

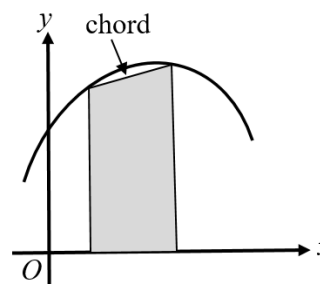
# Area under a graph

## A LEVEL LINKS

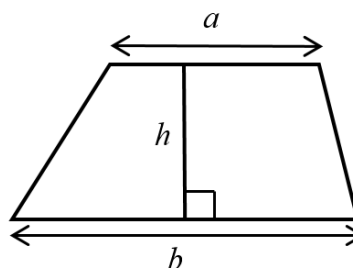
Scheme of work: 7b. Definite integrals and areas under curves

### Key points

- To estimate the area under a curve, draw a chord between the two points you are finding the area between and straight lines down to the horizontal axis to create a trapezium. The area of the trapezium is an approximation for the area under a curve.

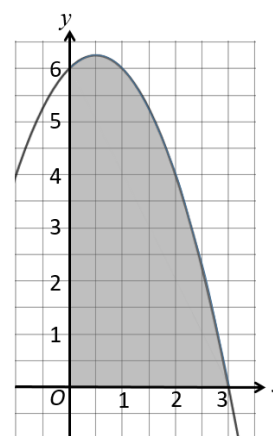


- The area of a trapezium =  $\frac{1}{2}h(a+b)$



### Examples

- Example 1** Estimate the area of the region between the curve  $y = (3 - x)(2 + x)$  and the  $x$ -axis from  $x = 0$  to  $x = 3$ . Use three strips of width 1 unit.



$x$	0	1	2	3
$y = (3 - x)(2 + x)$	6	6	4	0

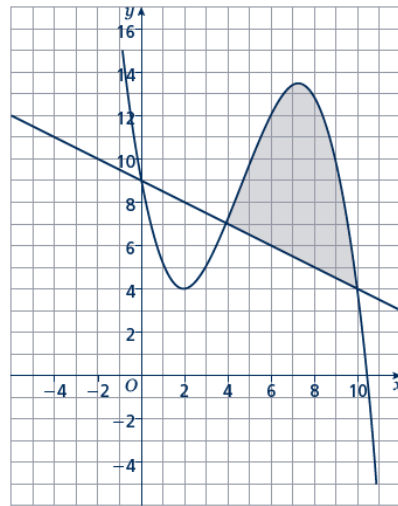
Trapezium 1:  
 $a_1 = 6 - 0 = 6$ ,  $b_1 = 6 - 0 = 6$   
 Trapezium 2:  
 $a_2 = 6 - 0 = 6$ ,  $b_2 = 4 - 0 = 4$   
 Trapezium 3:  
 $a_3 = 4 - 0 = 4$ ,  $b_3 = 0 - 0 = 0$

- Use a table to record the value of  $y$  on the curve for each value of  $x$ .
- Work out the dimensions of each trapezium. The distances between the  $y$ -values on the curve and the  $x$ -axis give the values for  $a$ .

*(continued on next page)*

$\frac{1}{2}h(a_1 + b_1) = \frac{1}{2} \times 1(6 + 6) = 6$ $\frac{1}{2}h(a_2 + b_2) = \frac{1}{2} \times 1(6 + 4) = 5$ $\frac{1}{2}h(a_3 + b_3) = \frac{1}{2} \times 1(4 + 0) = 2$ <p>Area = 6 + 5 + 2 = 13 units<sup>2</sup></p>	<p><b>3</b> Work out the area of each trapezium. <math>h = 1</math> since the width of each trapezium is 1 unit.</p> <p><b>4</b> Work out the total area. Remember to give units with your answer.</p>
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**Example 2** Estimate the shaded area.  
Use three strips of width 2 units.



<table border="1" style="width: 100%; border-collapse: collapse; margin-bottom: 10px;"> <tr><td style="text-align: center;"><b>x</b></td><td style="text-align: center;">4</td><td style="text-align: center;">6</td><td style="text-align: center;">8</td><td style="text-align: center;">10</td></tr> <tr><td style="text-align: center;"><b>y</b></td><td style="text-align: center;">7</td><td style="text-align: center;">12</td><td style="text-align: center;">13</td><td style="text-align: center;">4</td></tr> </table> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="text-align: center;"><b>x</b></td><td style="text-align: center;">4</td><td style="text-align: center;">6</td><td style="text-align: center;">8</td><td style="text-align: center;">10</td></tr> <tr><td style="text-align: center;"><b>y</b></td><td style="text-align: center;">7</td><td style="text-align: center;">6</td><td style="text-align: center;">5</td><td style="text-align: center;">4</td></tr> </table> <p>Trapezium 1: <math>a_1 = 7 - 7 = 0</math>, <math>b_1 = 12 - 6 = 6</math></p> <p>Trapezium 2: <math>a_2 = 12 - 6 = 6</math>, <math>b_2 = 13 - 5 = 8</math></p> <p>Trapezium 3: <math>a_3 = 13 - 5 = 8</math>, <math>b_3 = 4 - 4 = 0</math></p> $\frac{1}{2}h(a_1 + b_1) = \frac{1}{2} \times 2(0 + 6) = 6$ $\frac{1}{2}h(a_2 + b_2) = \frac{1}{2} \times 2(6 + 8) = 14$ $\frac{1}{2}h(a_3 + b_3) = \frac{1}{2} \times 2(8 + 0) = 8$ <p>Area = 6 + 14 + 8 = 28 units<sup>2</sup></p>	<b>x</b>	4	6	8	10	<b>y</b>	7	12	13	4	<b>x</b>	4	6	8	10	<b>y</b>	7	6	5	4	<ol style="list-style-type: none"> <li><b>1</b> Use a table to record <math>y</math> on the curve for each value of <math>x</math>.</li> <li><b>2</b> Use a table to record <math>y</math> on the straight line for each value of <math>x</math>.</li> <li><b>3</b> Work out the dimensions of each trapezium. The distances between the <math>y</math>-values on the curve and the <math>y</math>-values on the straight line give the values for <math>a</math>.</li> <li><b>4</b> Work out the area of each trapezium. <math>h = 2</math> since the width of each trapezium is 2 units.</li> <li><b>5</b> Work out the total area. Remember to give units with your answer.</li> </ol>
<b>x</b>	4	6	8	10																	
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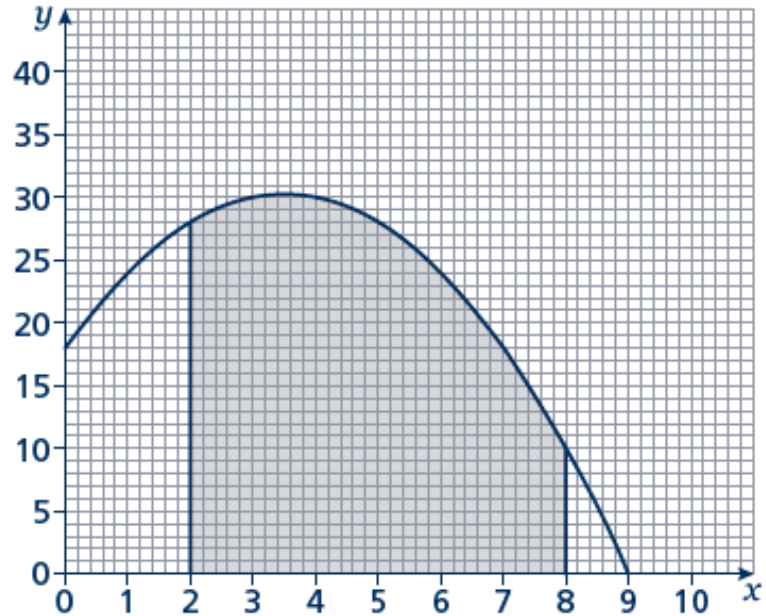
## Practice

- 1 Estimate the area of the region between the curve  $y = (5 - x)(x + 2)$  and the  $x$ -axis from  $x = 1$  to  $x = 5$ .  
Use four strips of width 1 unit.

**Hint:**

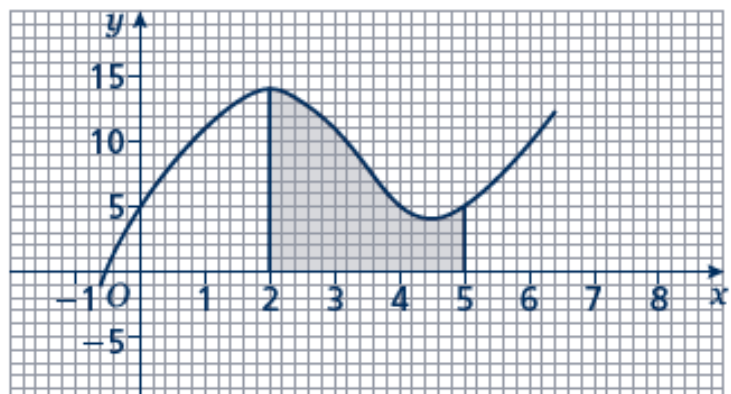
For a full answer, remember to include 'units<sup>2</sup>'.

- 2 Estimate the shaded area shown on the axes.  
Use six strips of width 1 unit.



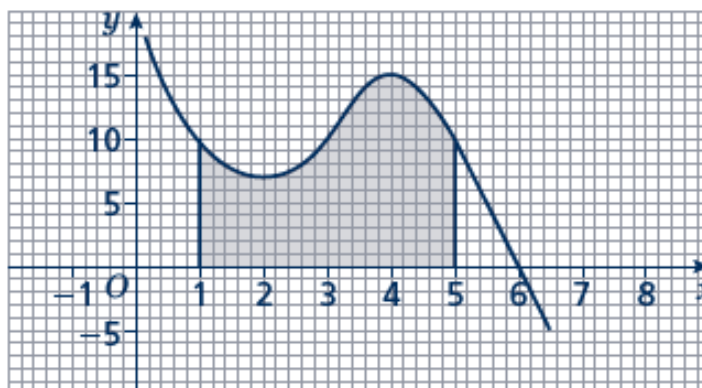
- 3 Estimate the area of the region between the curve  $y = x^2 - 8x + 18$  and the  $x$ -axis from  $x = 2$  to  $x = 6$ .  
Use four strips of width 1 unit.

- 4 Estimate the shaded area.  
Use six strips of width  $\frac{1}{2}$  unit.



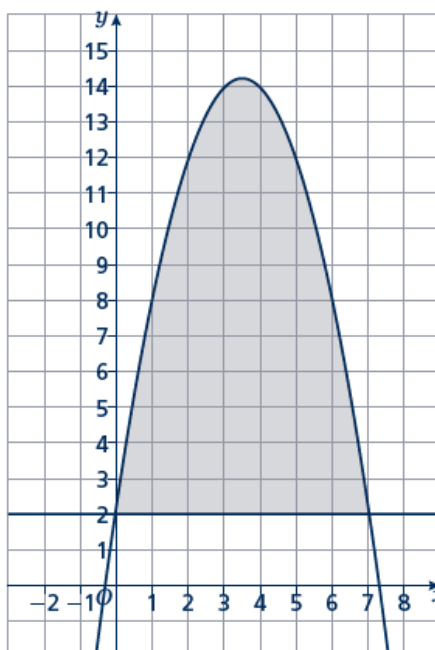
- 5 Estimate the area of the region between the curve  $y = -x^2 - 4x + 5$  and the  $x$ -axis from  $x = -5$  to  $x = 1$ .  
Use six strips of width 1 unit.

- 6 Estimate the shaded area.  
Use four strips of equal width.



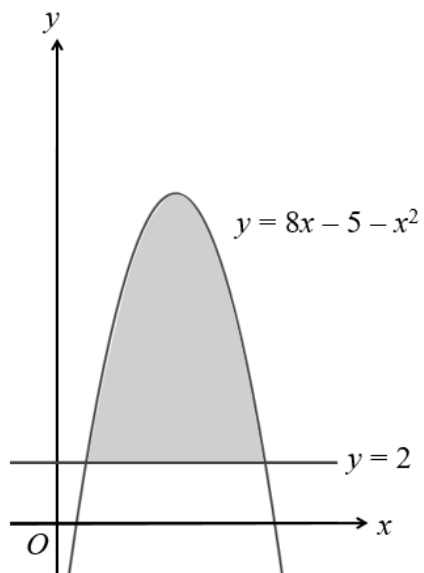
- 7 Estimate the area of the region between the curve  $y = -x^2 + 2x + 15$  and the  $x$ -axis from  $x = 2$  to  $x = 5$ .  
Use six strips of equal width.

- 8 Estimate the shaded area.  
Use seven strips of equal width.

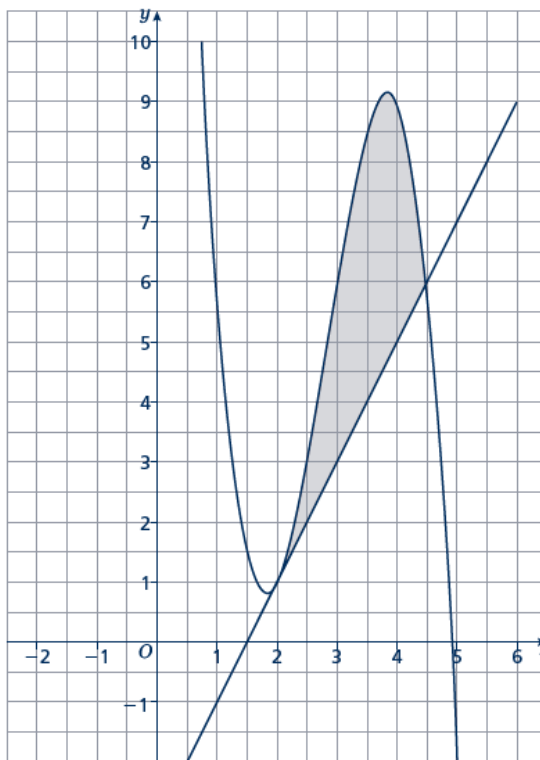


**Extend**

- 9 The curve  $y = 8x - 5 - x^2$  and the line  $y = 2$  are shown in the sketch. Estimate the shaded area using six strips of equal width.



- 10 Estimate the shaded area using five strips of equal width.



## Answers

1 34 units<sup>2</sup>

2 149 units<sup>2</sup>

3 14 units<sup>2</sup>

4  $25\frac{1}{4}$  units<sup>2</sup>

5 35 units<sup>2</sup>

6 42 units<sup>2</sup>

7  $26\frac{7}{8}$  units<sup>2</sup>

8 56 units<sup>2</sup>

9 35 units<sup>2</sup>

10  $6\frac{1}{4}$  units<sup>2</sup>