

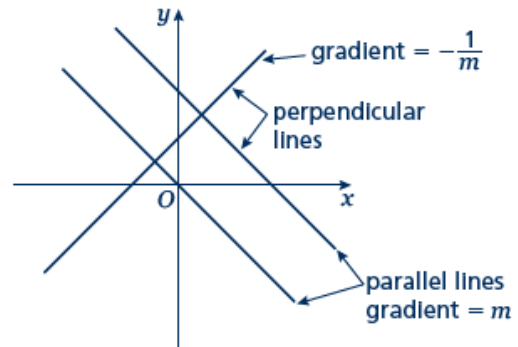
# Parallel and perpendicular lines

## A LEVEL LINKS

Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

### Key points

- When lines are parallel they have the same gradient.
- A line perpendicular to the line with equation  $y = mx + c$  has gradient  $-\frac{1}{m}$ .



### Examples

**Example 1** Find the equation of the line parallel to  $y = 2x + 4$  which passes through the point  $(4, 9)$ .

$y = 2x + 4$ $m = 2$ $y = 2x + c$ $9 = 2 \times 4 + c$ $9 = 8 + c$ $c = 1$ $y = 2x + 1$	<ol style="list-style-type: none"> <li>1 As the lines are parallel they have the same gradient.</li> <li>2 Substitute <math>m = 2</math> into the equation of a straight line <math>y = mx + c</math>.</li> <li>3 Substitute the coordinates into the equation <math>y = 2x + c</math></li> <li>4 Simplify and solve the equation.</li> <li>5 Substitute <math>c = 1</math> into the equation <math>y = 2x + c</math></li> </ol>
---	--

**Example 2** Find the equation of the line perpendicular to  $y = 2x - 3$  which passes through the point  $(-2, 5)$ .

$y = 2x - 3$ $m = 2$ $-\frac{1}{m} = -\frac{1}{2}$ $y = -\frac{1}{2}x + c$ $5 = -\frac{1}{2} \times (-2) + c$ $5 = 1 + c$ $c = 4$ $y = -\frac{1}{2}x + 4$	<ol style="list-style-type: none"> <li>1 As the lines are perpendicular, the gradient of the perpendicular line is <math>-\frac{1}{m}</math>.</li> <li>2 Substitute <math>m = -\frac{1}{2}</math> into <math>y = mx + c</math>.</li> <li>3 Substitute the coordinates <math>(-2, 5)</math> into the equation <math>y = -\frac{1}{2}x + c</math></li> <li>4 Simplify and solve the equation.</li> <li>5 Substitute <math>c = 4</math> into <math>y = -\frac{1}{2}x + c</math>.</li> </ol>
---	--

**Example 3** A line passes through the points (0, 5) and (9, -1).  
Find the equation of the line which is perpendicular to the line and passes through its midpoint.

$x_1 = 0, x_2 = 9, y_1 = 5 \text{ and } y_2 = -1$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 5}{9 - 0}$ $= \frac{-6}{9} = -\frac{2}{3}$ $-\frac{1}{m} = \frac{3}{2}$ $y = \frac{3}{2}x + c$ $\text{Midpoint} = \left( \frac{0+9}{2}, \frac{5+(-1)}{2} \right) = \left( \frac{9}{2}, 2 \right)$ $2 = \frac{3}{2} \times \frac{9}{2} + c$ $c = -\frac{19}{4}$ $y = \frac{3}{2}x - \frac{19}{4}$	<ol style="list-style-type: none"> <li><b>1</b> Substitute the coordinates into the equation <math>m = \frac{y_2 - y_1}{x_2 - x_1}</math> to work out the gradient of the line.</li> <li><b>2</b> As the lines are perpendicular, the gradient of the perpendicular line is <math>-\frac{1}{m}</math>.</li> <li><b>3</b> Substitute the gradient into the equation <math>y = mx + c</math>.</li> <li><b>4</b> Work out the coordinates of the midpoint of the line.</li> <li><b>5</b> Substitute the coordinates of the midpoint into the equation.</li> <li><b>6</b> Simplify and solve the equation.</li> <li><b>7</b> Substitute <math>c = -\frac{19}{4}</math> into the equation <math>y = \frac{3}{2}x + c</math>.</li> </ol>
--	--

## Practice

- 1** Find the equation of the line parallel to each of the given lines and which passes through each of the given points.
 

<b>a</b> $y = 3x + 1$ (3, 2)	<b>b</b> $y = 3 - 2x$ (1, 3)
<b>c</b> $2x + 4y + 3 = 0$ (6, -3)	<b>d</b> $2y - 3x + 2 = 0$ (8, 20)

- 2** Find the equation of the line perpendicular to  $y = \frac{1}{2}x - 3$  which passes through the point (-5, 3).

**Hint**

If  $m = \frac{a}{b}$  then the negative reciprocal  $-\frac{1}{m} = -\frac{b}{a}$

- 3** Find the equation of the line perpendicular to each of the given lines and which passes through each of the given points.
 

<b>a</b> $y = 2x - 6$ (4, 0)	<b>b</b> $y = -\frac{1}{3}x + \frac{1}{2}$ (2, 13)
<b>c</b> $x - 4y - 4 = 0$ (5, 15)	<b>d</b> $5y + 2x - 5 = 0$ (6, 7)



**Answers**

**1 a**  $y = 3x - 7$

**c**  $y = -\frac{1}{2}x$

**b**  $y = -2x + 5$

**d**  $y = \frac{3}{2}x + 8$

**2**  $y = -2x - 7$

**3 a**  $y = -\frac{1}{2}x + 2$

**c**  $y = -4x + 35$

**b**  $y = 3x + 7$

**d**  $y = \frac{5}{2}x - 8$

**4 a**  $y = -\frac{1}{2}x$

**b**  $y = 2x$

**5 a** Parallel**d** Perpendicular**b** Neither**e** Neither**c** Perpendicular**f** Parallel

**6 a**  $x + 2y - 4 = 0$

**b**  $x + 2y + 2 = 0$

**c**  $y = 2x$