

# Solving linear simultaneous equations using the elimination method

## A LEVEL LINKS

Scheme of work: 1c. Equations – quadratic/linear simultaneous

## Key points

- Two equations are simultaneous when they are both true at the same time.
- Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
- Make sure that the coefficient of one of the unknowns is the same in both equations.
- Eliminate this equal unknown by either subtracting or adding the two equations.

## Examples

**Example 1** Solve the simultaneous equations  $3x + y = 5$  and  $x + y = 1$

$\begin{array}{r} 3x + y = 5 \\ - \quad x + y = 1 \\ \hline 2x \quad = 4 \end{array}$ <p>So <math>x = 2</math></p> <p>Using <math>x + y = 1</math>  <math>2 + y = 1</math>            So <math>y = -1</math></p> <p>Check:            equation 1: <math>3 \times 2 + (-1) = 5</math> YES            equation 2: <math>2 + (-1) = 1</math> YES</p>	<p><b>1</b> Subtract the second equation from the first equation to eliminate the <math>y</math> term.</p> <p><b>2</b> To find the value of <math>y</math>, substitute <math>x = 2</math> into one of the original equations.</p> <p><b>3</b> Substitute the values of <math>x</math> and <math>y</math> into both equations to check your answers.</p>
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**Example 2** Solve  $x + 2y = 13$  and  $5x - 2y = 5$  simultaneously.

$\begin{array}{r} x + 2y = 13 \\ + \quad 5x - 2y = 5 \\ \hline 6x \quad = 18 \end{array}$ <p>So <math>x = 3</math></p> <p>Using <math>x + 2y = 13</math>  <math>3 + 2y = 13</math>            So <math>y = 5</math></p> <p>Check:            equation 1: <math>3 + 2 \times 5 = 13</math> YES            equation 2: <math>5 \times 3 - 2 \times 5 = 5</math> YES</p>	<p><b>1</b> Add the two equations together to eliminate the <math>y</math> term.</p> <p><b>2</b> To find the value of <math>y</math>, substitute <math>x = 3</math> into one of the original equations.</p> <p><b>3</b> Substitute the values of <math>x</math> and <math>y</math> into both equations to check your answers.</p>
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**Example 3** Solve  $2x + 3y = 2$  and  $5x + 4y = 12$  simultaneously.

$\begin{array}{r} (2x + 3y = 2) \times 4 \rightarrow 8x + 12y = 8 \\ (5x + 4y = 12) \times 3 \rightarrow \underline{15x + 12y = 36} \\ \phantom{(5x + 4y = 12) \times 3 \rightarrow} 7x = 28 \end{array}$ <p>So <math>x = 4</math></p> <p>Using <math>2x + 3y = 2</math>  <math>2 \times 4 + 3y = 2</math>  <math>8 + 3y = 2</math>  <math>3y = 2 - 8</math>  <math>3y = -6</math>  <math>y = -2</math></p> <p>Check:          equation 1: <math>2 \times 4 + 3 \times (-2) = 2</math> YES          equation 2: <math>5 \times 4 + 4 \times (-2) = 12</math> YES</p>	<p><b>1</b> Multiply the first equation by 4 and the second equation by 3 to make the coefficient of <math>y</math> the same for both equations. Then subtract the first equation from the second equation to eliminate the <math>y</math> term.</p> <p><b>2</b> To find the value of <math>y</math>, substitute <math>x = 4</math> into one of the original equations.</p> <p><b>3</b> Substitute the values of <math>x</math> and <math>y</math> into both equations to check your answers.</p>
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## Practice

Solve these simultaneous equations.

**1**  $4x + y = 8$   
 $x + y = 5$

**2**  $3x + y = 7$   
 $3x + 2y = 5$

**3**  $4x + y = 3$   
 $3x - y = 11$

**4**  $3x + 4y = 7$   
 $x - 4y = 5$

**5**  $2x + y = 11$   
 $x - 3y = 9$

**6**  $2x + 3y = 11$   
 $3x + 2y = 4$

# Solving linear simultaneous equations using the substitution method

## A LEVEL LINKS

**Scheme of work:** 1c. Equations – quadratic/linear simultaneous

**Textbook:** Pure Year 1, 3.1 Linear simultaneous equations

## Key points

- The substitution method is the method most commonly used for A level. This is because it is the method used to solve linear and quadratic simultaneous equations.

## Examples

**Example 4** Solve the simultaneous equations  $y = 2x + 1$  and  $5x + 3y = 14$

$5x + 3(2x + 1) = 14$ $5x + 6x + 3 = 14$ $11x + 3 = 14$ $11x = 11$ <p>So <math>x = 1</math></p> <p>Using <math>y = 2x + 1</math></p> $y = 2 \times 1 + 1$ <p>So <math>y = 3</math></p> <p>Check:</p> <p>equation 1: <math>3 = 2 \times 1 + 1</math>      YES</p> <p>equation 2: <math>5 \times 1 + 3 \times 3 = 14</math>    YES</p>	<ol style="list-style-type: none"> <li>Substitute <math>2x + 1</math> for <math>y</math> into the second equation.</li> <li>Expand the brackets and simplify.</li> <li>Work out the value of <math>x</math>.</li> <li>To find the value of <math>y</math>, substitute <math>x = 1</math> into one of the original equations.</li> <li>Substitute the values of <math>x</math> and <math>y</math> into both equations to check your answers.</li> </ol>
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**Example 5** Solve  $2x - y = 16$  and  $4x + 3y = -3$  simultaneously.

$y = 2x - 16$ $4x + 3(2x - 16) = -3$ $4x + 6x - 48 = -3$ $10x - 48 = -3$ $10x = 45$ <p>So <math>x = 4\frac{1}{2}</math></p> <p>Using <math>y = 2x - 16</math></p> $y = 2 \times 4\frac{1}{2} - 16$ <p>So <math>y = -7</math></p> <p>Check:</p> <p>equation 1: <math>2 \times 4\frac{1}{2} - (-7) = 16</math>      YES</p> <p>equation 2: <math>4 \times 4\frac{1}{2} + 3 \times (-7) = -3</math>    YES</p>	<ol style="list-style-type: none"> <li>Rearrange the first equation.</li> <li>Substitute <math>2x - 16</math> for <math>y</math> into the second equation.</li> <li>Expand the brackets and simplify.</li> <li>Work out the value of <math>x</math>.</li> <li>To find the value of <math>y</math>, substitute <math>x = 4\frac{1}{2}</math> into one of the original equations.</li> <li>Substitute the values of <math>x</math> and <math>y</math> into both equations to check your answers.</li> </ol>
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## Practice

Solve these simultaneous equations.

7  $y = x - 4$   
 $2x + 5y = 43$

8  $y = 2x - 3$   
 $5x - 3y = 11$

9  $2y = 4x + 5$   
 $9x + 5y = 22$

10  $2x = y - 2$   
 $8x - 5y = -11$

11  $3x + 4y = 8$   
 $2x - y = -13$

12  $3y = 4x - 7$   
 $2y = 3x - 4$

13  $3x = y - 1$   
 $2y - 2x = 3$

14  $3x + 2y + 1 = 0$   
 $4y = 8 - x$

## Extend

15 Solve the simultaneous equations  $3x + 5y - 20 = 0$  and  $2(x + y) = \frac{3(y - x)}{4}$ .

**Answers**

**1**  $x = 1, y = 4$

**2**  $x = 3, y = -2$

**3**  $x = 2, y = -5$

**4**  $x = 3, y = -\frac{1}{2}$

**5**  $x = 6, y = -1$

**6**  $x = -2, y = 5$

**7**  $x = 9, y = 5$

**8**  $x = -2, y = -7$

**9**  $x = \frac{1}{2}, y = 3\frac{1}{2}$

**10**  $x = \frac{1}{2}, y = 3$

**11**  $x = -4, y = 5$

**12**  $x = -2, y = -5$

**13**  $x = \frac{1}{4}, y = 1\frac{3}{4}$

**14**  $x = -2, y = 2\frac{1}{2}$

**15**  $x = -2\frac{1}{2}, y = 5\frac{1}{2}$