

Surds and rationalising the denominator

A LEVEL LINKS

Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- A surd is the square root of a number that is not a square number, for example $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, etc.
- Surds can be used to give the exact value for an answer.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- To rationalise the denominator means to remove the surd from the denominator of a fraction.
- To rationalise $\frac{a}{\sqrt{b}}$ you multiply the numerator and denominator by the surd \sqrt{b}
- To rationalise $\frac{a}{b + \sqrt{c}}$ you multiply the numerator and denominator by $b - \sqrt{c}$

Examples

Example 1 Simplify $\sqrt{50}$

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| $\begin{aligned}\sqrt{50} &= \sqrt{25 \times 2} \\ &= \sqrt{25} \times \sqrt{2} \\ &= 5 \times \sqrt{2} \\ &= 5\sqrt{2}\end{aligned}$ | <ol style="list-style-type: none"> 1 Choose two numbers that are factors of 50. One of the factors must be a square number 2 Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ 3 Use $\sqrt{25} = 5$ |
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Example 2 Simplify $\sqrt{147} - 2\sqrt{12}$

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| $\begin{aligned}\sqrt{147} - 2\sqrt{12} \\ &= \sqrt{49 \times 3} - 2\sqrt{4 \times 3} \\ \\ &= \sqrt{49} \times \sqrt{3} - 2\sqrt{4} \times \sqrt{3} \\ &= 7 \times \sqrt{3} - 2 \times 2 \times \sqrt{3} \\ &= 7\sqrt{3} - 4\sqrt{3} \\ &= 3\sqrt{3}\end{aligned}$ | <ol style="list-style-type: none"> 1 Simplify $\sqrt{147}$ and $2\sqrt{12}$. Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number 2 Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ 3 Use $\sqrt{49} = 7$ and $\sqrt{4} = 2$ 4 Collect like terms |
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Example 3 Simplify $(\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2})$

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| $ \begin{aligned} &(\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2}) \\ &= \sqrt{49} - \sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7} - \sqrt{4} \\ &= 7 - 2 \\ &= 5 \end{aligned} $ | <ol style="list-style-type: none"> 1 Expand the brackets. A common mistake here is to write $(\sqrt{7})^2 = 49$ 2 Collect like terms: $\begin{aligned} &-\sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7} \\ &= -\sqrt{7}\sqrt{2} + \sqrt{7}\sqrt{2} = 0 \end{aligned}$ |
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Example 4 Rationalise $\frac{1}{\sqrt{3}}$

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| $ \begin{aligned} \frac{1}{\sqrt{3}} &= \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{1 \times \sqrt{3}}{\sqrt{9}} \\ &= \frac{\sqrt{3}}{3} \end{aligned} $ | <ol style="list-style-type: none"> 1 Multiply the numerator and denominator by $\sqrt{3}$ 2 Use $\sqrt{9} = 3$ |
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Example 5 Rationalise and simplify $\frac{\sqrt{2}}{\sqrt{12}}$

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| $ \begin{aligned} \frac{\sqrt{2}}{\sqrt{12}} &= \frac{\sqrt{2}}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}} \\ &= \frac{\sqrt{2} \times \sqrt{4 \times 3}}{12} \\ &= \frac{2\sqrt{2}\sqrt{3}}{12} \\ &= \frac{\sqrt{2}\sqrt{3}}{6} \end{aligned} $ | <ol style="list-style-type: none"> 1 Multiply the numerator and denominator by $\sqrt{12}$ 2 Simplify $\sqrt{12}$ in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number 3 Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ 4 Use $\sqrt{4} = 2$ 5 Simplify the fraction: $\frac{2}{12}$ simplifies to $\frac{1}{6}$ |
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Example 6 Rationalise and simplify $\frac{3}{2+\sqrt{5}}$

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| $\frac{3}{2+\sqrt{5}} = \frac{3}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$ $= \frac{3(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})}$ $= \frac{6-3\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-5}$ $= \frac{6-3\sqrt{5}}{-1}$ $= 3\sqrt{5}-6$ | <p>1 Multiply the numerator and denominator by $2-\sqrt{5}$</p> <p>2 Expand the brackets</p> <p>3 Simplify the fraction</p> <p>4 Divide the numerator by -1 Remember to change the sign of all terms when dividing by -1</p> |
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Practice

1 Simplify.

a $\sqrt{45}$

c $\sqrt{48}$

e $\sqrt{300}$

g $\sqrt{72}$

b $\sqrt{125}$

d $\sqrt{175}$

f $\sqrt{28}$

h $\sqrt{162}$

Hint

One of the two numbers you choose at the start must be a square number.

2 Simplify.

a $\sqrt{72} + \sqrt{162}$

c $\sqrt{50} - \sqrt{8}$

e $2\sqrt{28} + \sqrt{28}$

b $\sqrt{45} - 2\sqrt{5}$

d $\sqrt{75} - \sqrt{48}$

f $2\sqrt{12} - \sqrt{12} + \sqrt{27}$

Watch out!

Check you have chosen the highest square number at the start.

3 Expand and simplify.

a $(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})$

c $(4 - \sqrt{5})(\sqrt{45} + 2)$

b $(3 + \sqrt{3})(5 - \sqrt{12})$

d $(5 + \sqrt{2})(6 - \sqrt{8})$

4 Rationalise and simplify, if possible.

a $\frac{1}{\sqrt{5}}$

b $\frac{1}{\sqrt{11}}$

c $\frac{2}{\sqrt{7}}$

d $\frac{2}{\sqrt{8}}$

e $\frac{2}{\sqrt{2}}$

f $\frac{5}{\sqrt{5}}$

g $\frac{\sqrt{8}}{\sqrt{24}}$

h $\frac{\sqrt{5}}{\sqrt{45}}$

5 Rationalise and simplify.

a $\frac{1}{3-\sqrt{5}}$

b $\frac{2}{4+\sqrt{3}}$

c $\frac{6}{5-\sqrt{2}}$

Extend

6 Expand and simplify $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})$

7 Rationalise and simplify, if possible.

a $\frac{1}{\sqrt{9}-\sqrt{8}}$

b $\frac{1}{\sqrt{x}-\sqrt{y}}$

Answers

1 a $3\sqrt{5}$
 c $4\sqrt{3}$
 e $10\sqrt{3}$
 g $6\sqrt{2}$

b $5\sqrt{5}$
 d $5\sqrt{7}$
 f $2\sqrt{7}$
 h $9\sqrt{2}$

2 a $15\sqrt{2}$
 c $3\sqrt{2}$
 e $6\sqrt{7}$

b $\sqrt{5}$
 d $\sqrt{3}$
 f $5\sqrt{3}$

3 a -1
 c $10\sqrt{5}-7$

b $9-\sqrt{3}$
 d $26-4\sqrt{2}$

4 a $\frac{\sqrt{5}}{5}$
 c $\frac{2\sqrt{7}}{7}$
 e $\sqrt{2}$
 g $\frac{\sqrt{3}}{3}$

b $\frac{\sqrt{11}}{11}$
 d $\frac{\sqrt{2}}{2}$
 f $\sqrt{5}$
 h $\frac{1}{3}$

5 a $\frac{3+\sqrt{5}}{4}$

b $\frac{2(4-\sqrt{3})}{13}$

c $\frac{6(5+\sqrt{2})}{23}$

6 $x-y$

7 a $3+2\sqrt{2}$

b $\frac{\sqrt{x}+\sqrt{y}}{x-y}$